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Observations on the Anomalous Behaviour of Delicate Balances, and an Account of Devices for Increasing Accuracy in Weighings

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XI. *Observations on the Anomalous Behaviour of Delicate Balances, and an Account of Devices for Increasing Accuracy in Weighings.*

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Introductory.

IN this communication it is desired to draw attention to certain difficulties which were encountered during the standardization of a set of weights, and to show how they were investigated and overcome.

The weights were compared by the combined methods of vibration and reversal, the sensibility of the balance being determined for each particular load either immediately before or after the completion of a comparison. The usual custom of observing the pointer through a telescope was also followed. The limits of the pointer's excursions to the right and left of the resting-point (R.P.) were estimated in tenths of a scale division.

The standardizations were effected by means of a Bunge short-beam balance, and, in order to obtain a check upon the indicated values, it was decided to re-compare several of the weights with the aid of an Oertling balance having a long beam. It was then discovered that the two balances gave slightly, but distinctly, different values for the several weights; it was likewise found that values differing from each other to a very small extent were generally indicated in each succeeding comparison with either balance.

On referring to numerous papers dealing with atomic weight determinations, and other work involving very accurate weighing, it was found that many observers had experienced similar difficulties.

MILLER, in his "Determination of the Standard Pound,"* alludes to certain progressive changes in the resting-point of his balances, and suggests causes to which they may be due.

During his observations for the determination of the "Mean Density of the Earth," POYNTING† was troubled by the shifting to and fro of the resting-point. THORPE, in his paper on the atomic weight of titanium,‡ draws particular attention to the inexplicable behaviour of his Oertling short-beam balance. When determining the atomic weight of boron, ABRAHALL§ observed a "slow change of the zero-point, continuous throughout."

It would appear that BAILEY and LAMB|| were more fortunate than some other observers have been. They state that during their determinations of the atomic weight of palladium, "the extreme variation of the zero-point did not exceed 0·3 division in two weeks." The sensibility (11·1) of their balance was, however, somewhat low.

DEAN¶ "found that the weighings could be made quite easily to a tenth of a milligramme, provided that the final weighing was not made until half-an-hour after the tube and weights had been placed upon the pans of the balance. . . . . That the half-hour interval needed was not on account of hygroscopic moisture absorbed from the air by the material is shown by the fact that the weighings taken immediately were always *greater* than those taken later."

For the re-determination of the atomic weight of nitrogen, GRAY** used a balance by SARTORIUS. The instrument was placed in a cellar and screened by a tin box.

* 'Phil. Trans.,' 1856, p. 765.

† 'Mean Density of the Earth,' pp. 99 and 156. Griffin and Co., 1894.

‡ 'J. C. S.,' 1885, p. 116.

§ 'J. C. S.,' 1892, p. 661.

|| 'J. C. S.,' 1892, p. 752.

¶ 'J. C. S.,' 1900, p. 128.

** 'J. C. S.,' 1905, p. 1608.

This device would assist very considerably in overcoming some of the difficulties likely to arise.

In carrying out their experiments to ascertain whether change in temperature affects weight, POYNTING and PHILLIPS* observed a temporary apparent change in weight equal to 0·034 mgr. per 1° C. This change they ascribed to the effects produced upon the balance by convection currents and radiant energy.

DIXON and EDGAR† placed their balance in a cellar and allowed half-an-hour to elapse after adjusting the weights before the final weighings were made.

HICKS‡ has drawn attention to BAILY's results for the mean density of the Earth, and has shown that if they are arranged in the order of the temperature of the apparatus they decrease in value as the temperature increases. We agree with POYNTING and PHILLIPS, and believe that the more or less uniform variations noticed by HICKS are due to corresponding variations in the temperature of BAILY's apparatus, and to these alone.

It is not improbable that the differences in the values obtained for the mean density of the Earth by JOLLY,§ in Munich, RICHARZ and KRIGAR-MENSEL,‖ at Spandau, and by POYNTING, in his laboratory at the Mason College, are at least partly attributable to unknown and unexpected changes which took place within some of the balances used by those observers.

The supposed variations in weight observed by LANDOLT¶ during certain chemical changes may, we think, be almost entirely ascribed to fluctuations occurring within the beam of the instrument used for weighing.

Preliminary Observations and Methods.

When two weights are being compared, it is, we believe, usual to note the limits of not more than five excursions of the pointer: three observations being taken on the one, and two on the other side of the resting-point. From the data thus obtained, the value of the resting-point (R.P.) is deduced.

Now, it would appear to be quite reasonable to suppose that for a given weight, a repetition of the observations would lead to an almost, if not quite, identical value for the R.P. Therefore, with the view of testing the truth of this supposition, an extended series of experiments, with first-grade balances of various types and by different makers, was planned.

When the beam of a highly finished balance is made to oscillate, the rate of the

* 'Proc. Roy. Soc.,' 1905, p. 451.

† 'Phil. Trans.,' A, 1906, p. 185.

‡ 'Proc. Camb. Phil. Soc.,' vol. V., p. 156.

§ 'WIED. Ann.,' XIV., 1881, pp. 331–355.

‖ 'Abhand. der Königl. Preuss. Akad.,' Berlin, 1898.

¶ 'Preuss. Ak. Wiss.,' Berlin, Sitz. Ber. VIII., 1906.

decay in the amplitude is very low; hence it is not difficult for an observer, using a telescope, to obtain data for the determination of the R.P. during any period of the 8 to 10 minutes which immediately follow the release of the beam. By continuous observation we are, therefore, able to accumulate sufficient data for some 18 or more independent determinations of the R.P. before the amplitude of the oscillations becomes too small to yield trustworthy results.

In calculating the R.P. from the data, we may take successive pointer readings in groups of three or five. As it appeared that the use of small, rather than that of large, groups would yield the desired information with the greatest degree of certainty, it was decided to calculate the R.Ps. generally from groups of three observations.

We here give typical data obtained by employing a No. 1F Bunge balance, the beam of which was solid and 13 cm. in length, and a load of 20 gr. We may mention that this is but one taken from some scores of similar examples.

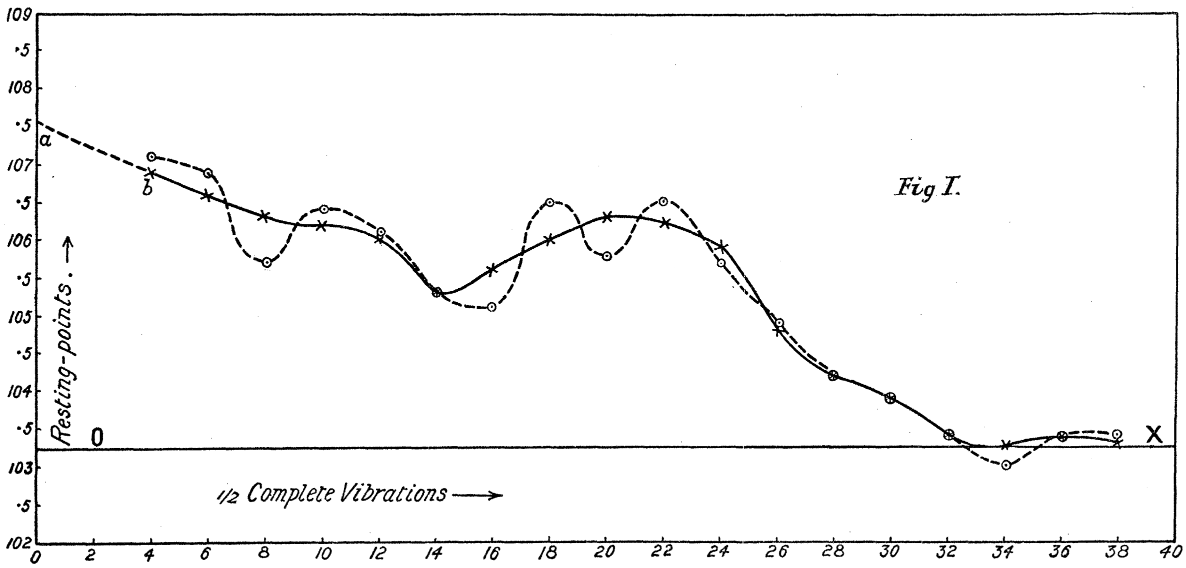
TABLE I.

Observa-tions.	Pointer readings.	Observa-tions.	Pointer readings.	Series I. R.Ps. calculated from groups of 3 observations.			Series II. Mean R.Ps. calculated from groups of 5 observations.	Differences between means.
				α.	β.	Means.		
1	10	2	199	106·8	107·3	107·1	106·9	− 0·2
3	19	4	192	7·3	6·5	6·9	6·6	− ·3
5	26	6	182	5·5	5·8	5·7	6·3	+ ·6
7	32	8	177	6·3	6·5	6·4	6·2	− ·2
9	39	10	171	6·3	5·8	6·1	6·0	− ·1
11	44	12	164	5·3	5·3	5·3	5·3	± ·0
13	49	14	159	4·8	5·3	5·1	5·6	+ ·5
15	52	16	158	6·5	6·5	6·5	6·0	− ·5
17	58	18	152	5·5	6·0	5·8	6·3	+ ·5
19	60	20	152	6·5	6·5	6·5	6·2	− ·3
21	62	22	150	6·0	5·3	5·7	5·9	+ ·2
23	62	24	147	5·3	4·5	4·9	4·8	− ·1
25	65	26	141	4·0	4·3	4·2	4·2	± ·0
27	69	28	138	4·0	3·8	3·9	3·9	± ·0
29	71	30	135	3·5	3·3	3·4	3·4	± ·0
31	73	32	132	3·0	3·0	3·0	3·3	+ ·3
33	75	34	130	3·3	3·5	3·4	3·4	± ·0
35	78	36	128	3·5	3·3	3·4	3·3	+ ·1
37	80	38	125	2·8		2·8		
39	81							
Maximum difference						4·3	3·6	

From the above data two series of values for the R.P. were calculated: Series I. being derived from groups of three, and Series II. from groups of five, observations. We here state, more precisely, how the different and successive values for the R.P., given under (α) and (β) in Table I., were deduced.

After releasing the beam, two complete vibrations were allowed to take place before commencing the observations. Some 30 to 40 readings of the extreme positions of the pointer then followed, and the data were utilised thus: Observations 1, 2, and 3 were taken and the first value for the R.P. given under (α) calculated; the second value was calculated from observations 3, 4, and 5; the third value from observations 5, 6, and 7; and so on. Similarly, the values enumerated under (β) were deduced from the groups 2, 3, and 4; 4, 5, and 6; 6, 7, and 8, &c.; the final values for the R.Ps. being those obtained by taking the means of (α_1) and (β_1); (α_2) and (β_2); (α_3) and (β_3); &c.

The mean values given in Column 8 result from the application of the same method to overlapping groups of five observations. Plotting the successive values of the R.P. along the Y-axis, and the corresponding times along the X-axis, we obtain the curves shown in Fig. I.



The more sinuous and dotted line curve is obtained by making use of the R.Ps. calculated from groups of three observations of the pointer; the other curve is a representation of the values derived from calculations based upon groups of five.

The type of curve here presented is quite unlike that which we should expect to obtain. It will be observed that, although the general trend of the two curves is identical, the first named supplies greater detail. Towards the end of the experiment, however, the two almost coalesce, and, just before the conclusion of the observations, the R.P. becomes almost constant, and approaches an ideal value which is represented by the curve OX. The portion *ab* was derived by extrapolating.

Very similar curves were obtained as the result of experimenting with loads ranging from 10 to 200 gr., and, with only one or two exceptions, the other balances which were tested in this research behaved in a similar manner.

Of the Conditions under which the Experiments were Conducted.

In the preliminary experiments the balances were used under the more or less favourable conditions which obtain in a laboratory where quantitative analysis is carried on. But all subsequent and final experiments, from which our conclusions were drawn, were conducted in a room having a N.E. aspect, and possessing, in general, a very steady temperature; and in order to maintain as constant a temperature as possible within the balance case, a temporary cupboard was, in every instance, built up round the instrument. This cupboard was constructed of sheets of thick asbestos and plate glass, the interspaces being, to a large extent, loosely packed with cotton wool.

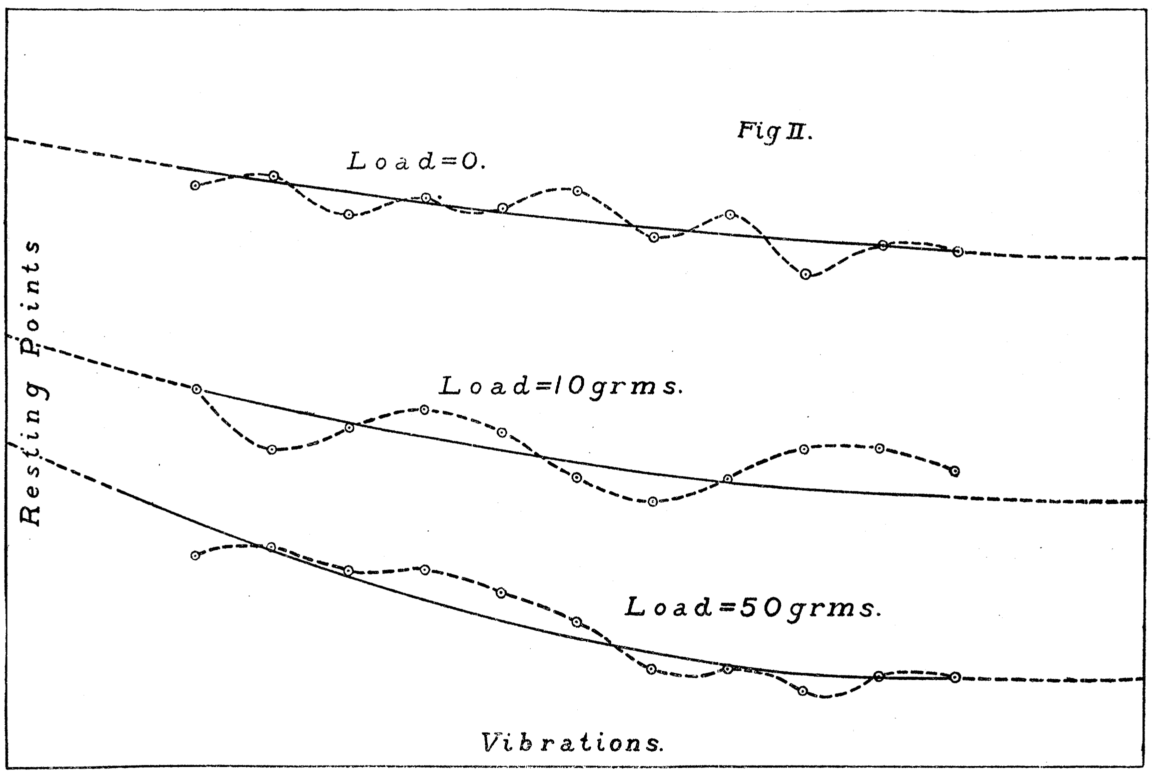
The temperature within was indicated by two sensitive thermometers made of Jena "normal" glass. These thermometers, previously standardized, were placed parallel to the length of the beam, the bulb of the one being brought as close as possible to the end of the left-hand, or L, arm, whilst that of the other was similarly placed with regard to the right-hand, or R, arm. By means of a telescope the temperatures were estimated to 1/100° C.

A little experience showed that a distinct advantage was gained by illuminating the room with yellow light only.

The monochromatic light was conveniently obtained by making use of a 6-litre flask, which was nearly filled with a solution of potassium chromate of suitable strength. A wide glass tube, closed at the lower end, was introduced into the flask, and inside this was placed a glow-lamp of cylindrical form. The flask was then supported in a position immediately opposite the centre of the balance case, and at a distance of about one metre. This method of illuminating the scale proved highly satisfactory, chiefly on account of the ease with which a sharp image of the pointer could be obtained in the reading telescope. As the heat rays emitted by the lamp were absorbed by the chromate solution, the balance was not affected by those disturbing influences which result from the near presence of a lamp under ordinary conditions. Finally, the screening of the window to exclude extraneous light helped very materially to maintain for short periods the desired uniform and constant temperature.

Some Experiments with a Circular Beam.

One of the first instruments experimented with under the conditions just described was a balance having a circular beam. Numerous tests with loads varying from that of the pans alone (commonly termed zero load) to the maximum of 200 gr. were conducted. The results obtained when the loads were respectively 0, 10, and 50 gr. are shown graphically in Fig. II. They are typical of all the other loads employed.



The sinuous and dotted line curves represent the fluctuations in the resting-point (R.P.); the others indicate very approximately the mean rate of change in the R.P., from a maximum to a somewhat lower and normal value. In these particular cases we observe that the variations in the R.P. are not so strongly marked as are those which we have represented in Fig. I.

The dotted line graphs suggest, more or less markedly, curves of sines. We also note that an increase in the load produces in the first portions of the curve a greater degree of steepness. But in these, as in some hundreds of similar cases which were observed, the R.P. ultimately tends to assume a constant and ideal value.

When the decay in the amplitude of the oscillations has become so great that reliable readings of the pointer are no longer possible, one of the pans may be gently tapped with its arrestor, and an additional set of observations taken. Plotting the curve as before, we find that it still continues sinuous in form, but that the axis of the sinuosities now travels almost horizontally. If, however, the experiment is repeated after the beam has been arrested for a short period then the results obtained are similar to those shown in Fig. II.

Possible Causes of the Observed Irregularities.

From a study of the data available after experimenting for a considerable time with one long and five short beam balances, we were led to conclude that the changes in the R.P. were, although small, very real; and that they were probably due to one or more of the following causes:—

(1) A slipping to and fro of the central knife-edge upon its plane whilst the beam is executing vibrations having a maximum amplitude.

(2) A slight but continued differential flexure in the two arms lasting for some minutes after the release of the beam.

(3) A very small lateral displacement of one or more of the knife-edges, due to the yielding of the screws or blocks which hold them in position.

(4) Unequal lengthening of the two arms by contact with the warm air currents ascending from the experimenter's hand during the process of loading the pans.

We proceed to consider each of these causes in detail.

(1) *Side-slip.*

That the changes observed in the R.P. were at least partly due to the central knife-edge slipping to and fro upon its supporting plane appeared to be rendered still more probable by a discovery made during an examination of the Oertling balance. This particular balance has been in use for nearly forty years; it is still very sensitive and highly satisfactory. A careful search revealed the existence of a fine rectilinear groove cut along the middle of the central plane where it comes into contact with the knife-edge. Oft repeated side-slip would easily account for the formation of such a groove.

Admitting the possibility of side-slip, we may determine what effects, if any, this would have upon the value obtained for the R.P.

In the case of *strictly uniform* side-slip, purely theoretical considerations show that its effect would merely be to increase the amplitude of the vibrations of the pointer without in any way changing the value for the R.P.

With regard to the possible existence of non-uniform side-slip, it is, we think, only necessary to here state that certain experiments based upon an optical method were conducted with a Bunge beam; and that the results obtained conclusively proved that if any side-slip whatever took place its magnitude was something less than 0·0001 mm. So small a value is for our purposes quite insignificant, and need not be further considered.

We are, therefore, justified in concluding that the beam, even when executing vibrations of exceptionally large amplitude, is practically free from side-slip, whether regular or otherwise. It likewise follows that the relatively large undulatory movement of the R.P. must be due to some other cause.

(2) *Of Flexure.* (*a*) *General Effects.*

It is well known that the sensibility of a balance almost invariably changes with the applied load. The changes may be attributed to the flexure of the beam: and, according to the view most generally held, such flexure would, for all practical purposes, take place uniformly and almost, if not quite, instantaneously. If this view is correct, it follows, provided all other conditions which can affect the position of the R.P. remain constant, (1) that for any given load both the sensibility and the R.P. will remain invariable from the moment the beam is released; and (2) that any observed deviations will fall within the limits of certain experimental errors, the magnitude of which can be estimated with a fair degree of accuracy. Such errors would naturally assume positive values in some cases, and approximately equal negative values in others; the curve representing the mean result of a series of determinations of the R.P. would be, if drawn after the manner adopted in Figs. I. and II., a horizontal straight line.

If, however, the flexure is neither uniform throughout the beam, nor completed within an inappreciable time after releasing the beam, we should expect to find some such effects as those already noticed. Want of strict uniformity of flexure in the two arms would lead to a shifting of the R.P. to the right or left of a truly normal position. Uniform and equal flexure of both arms, whether instantaneous and complete or not, would affect the sensibility only.

(*b*) *Of the Nature and Degree of Flexure.*

With the object of discovering whether the flexure of the beam, consequent upon the application of any given load, is instantaneous and complete or not, three beams were tried, namely, a Becker solid beam, a Bunge solid beam, and a circular beam.

In each case the terminal knife-edges of the beam were suitably supported upon glass blocks resting upon a rigid bench. The pan for carrying the required weights was suspended from the central knife-edge by means of a stout wire terminating above in two hooks, between which the beam was inserted. The depressions of the centre of the beam, resulting from the application of various loads, were measured with the aid of an optical lever. With the apparatus used, the smallest depression that could be observed with certainty was equal to 0·0004 mm.

The following experiments were made:—(1) *Becker beam.*

Experiment I.—A load of 400 gr. was kept in the pan for 13 hours. The load was then removed and the position of the image upon the scale at once noted. Ten minutes later the deflection was still the same.

Experiment II.—The pan was loaded with 900 gr. and the deflection immediately read. During a lapse of 10 minutes the deflection remained unchanged.

Experiment III.—The load (Experiment II.) was now removed and the deflection reading taken. No change in its value occurred during the ensuing 5 minutes.

Experiment IV.—One arm of the loaded beam was gently warmed. This produced an *increase* in the deflection. As the beam cooled, the deflection assumed its original value. When the other arm was warmed the deflection *decreased;* but as the arm cooled, the deflection assumed, as before, its normal value.

The first three experiments lead us to infer (a) that even when the beam is kept loaded for an exceptionally long period, on removing the stress, the recovery from the strain is, for all practical purposes, instantaneous; and (b) the moment the beam is loaded, the flexure reaches a maximum.

The fourth experiment is interesting because it indicates the necessity of maintaining the whole of the beam at some uniform temperature.

The beam was also tested with gradually increasing loads ranging from 50 to 1000 gr. In every case the flexure reached its maximum value at once, and the recovery after unloading was equally rapid.

The experiments just described were repeated with the Bunge and circular beams. The results (Table II.) were very similar in the case of all three beams.

TABLE II.

	Depressions.		
Load.	Becker beam.	Bunge beam.	Circular beam.
gr.	mm.	mm.	mm.
50	0·0021	0·0007	0·0015
100	0·0053	0·0015	0·0020
200	0·0105	0·0030	0·0038
400	0·0180	—	—
500	0·0250	0·0078	0·0117
700	0·0336	—	—
1000	0·0462	0·0169	0·0245

The results obtained with the Becker beam are best represented by a straight-line graph. The Bunge beam behaved in a very uniform manner, and all the points lie upon a smooth curve; but the circular beam showed some irregularity. As this beam was subsequently found to be readily affected by changes in temperature, the observed irregularities are probably to a large extent due to this cause.

(c) *Of the Possibility of Differential Flexure.*

Although the experiments cited in the preceding section support the view that any flexure in a loaded beam is instantaneous, they do not enable us to decide whether such flexure is equally distributed over the two arms or not. To determine this, the following additional experiments were made:—

The beam was supported as before; but, instead of using the optical lever, a small concave mirror was secured to either end of the beam, and the lamp and scale arranged as in the former case.

Two series of experiments were made: the one with the Becker, and the other with the circular beam. Having attached the mirror to the extremity of the L arm of the beam, various loads were placed in the pan, and the resulting deflections noted. The mirror was now removed from the L arm and secured to the extremity of the R arm. Reversing the beam in order to interchange the positions occupied by the terminal knife-edges, the experiments with the several loads were repeated. For equal loads the values obtained for the flexure of each arm were in every case identical.

Similar experiments conducted with the circular beam also gave concordant results for the two arms. From these experiments we conclude that if the arms exhibit a differential flexure when the beam is loaded, its limiting value is less than 0·0004 mm. If, however, there is any difference in the flexure, the effective length of the arm which bends to the greatest extent will be less than that of the more rigid one; and consequently the R.P. will be shifted in the direction of the shortened arm.

It can be shown that the accuracy of weighing is not appreciably affected even when a possible inequality in the rigidity of the arms results in a difference in the depressions equal to 0·0004 mm.

I am indebted to Mr. H. H. Hilton, formerly Fellow of Magdalen College, Oxford, for the following proof of the accuracy of this view.

Let us assume the arms to be of equal length, 70 mm. Suppose that when the pans are loaded with weights, W, w, the arms are depressed through small angles, α, β, respectively, from the horizontal, owing to a lack of perfect rigidity in the arms.

Then, if equilibrium exists,

$$\mathrm{W} \cos \alpha = w \cos \beta,$$

therefore

$$\mathrm{W} - w = \mathrm{W}\,(1 - \cos \alpha \sec \beta)$$

$$= \tfrac{1}{2}\mathrm{W}\,(\alpha - \beta)\,(\alpha + \beta) \text{ approximately.}$$

Now, in the experiments described above, the difference of depression of the two ends of the beam was never more than 0·0004 mm. even when W and w = 200 gr.; therefore $(\alpha-\beta)$ was never more than 0·0004/70 = 0·0000057; while $(\alpha+\beta)$ was never more than 0·006. Therefore W$-w$ was never more than $\frac{1}{2} \times 200 \times 0{\cdot}0000057 \times 0{\cdot}006$ = 0·000004 gr., which is a negligible quantity.

After reviewing the whole of the available evidence, we were led to conclude that although a difference in the flexure of the two arms of a loaded beam is possible and probable, yet its magnitude, even in the most marked case, is so insignificant that it could produce no appreciable error.

Dismissing, therefore, the possibility of the introduction of any significant error from this cause, we must seek for some other explanation of the observed changes which so often take place in the R.P. during a short period immediately following the release of the beam.

That some change which is equivalent to the lengthening of one arm or the shortening of the other actually occurs appears to be established beyond doubt, not only by the experiments instituted by myself, but also by independent experiments conducted by several observers researching in this laboratory and using other balances.

(3) *Of the Possibility of the Lateral Displacement of a Knife-edge.*

So far as the writer is aware, there now remain only two possible ways of accounting for the unexpected and anomalous behaviour exhibited by the beam immediately after its release, viz. :—(1) the lateral displacement, or its equivalent, of a knife-edge, and (2) the changes in the temperature of the beam during loading. In this section we confine ourselves to a consideration of the first-named possibility.

On loading a balance the resultant strains set up within it will necessarily be shared by the beam as a whole; they would naturally tend to bring about some slight movements among the blocks and somewhat numerous screws used for securing the knife-edges. The stresses would probably exert the greatest effect when the inclination of the beam during its oscillations is a maximum, for at that moment the bases of the three agate prisms would lie upon planes the inclination of which would also be a maximum, and the side-thrust exerted by the horizontal planes which are in contact with the knife-edges would then be as large as possible.

Now we should expect any effects produced in this way to show themselves most markedly at first, and then to die away as the knife-edges accommodated themselves to the new conditions imposed by the load. Generally it would be exceedingly difficult, if not impossible, to obtain any direct evidence other than that which may be gathered from the data for the R.P., which would confirm or disprove the truth of this supposition. But upon two occasions, when experimenting with a triangular beam to obtain data for an R.P. curve, we were so fortunate as to actually observe a series of minute joltings of the pointer. We believe that these were caused by sudden yieldings of some one or more of the knife-edge screws. The changes in the R.P. resulting from these joltings are indicated in curve α, Fig. III.

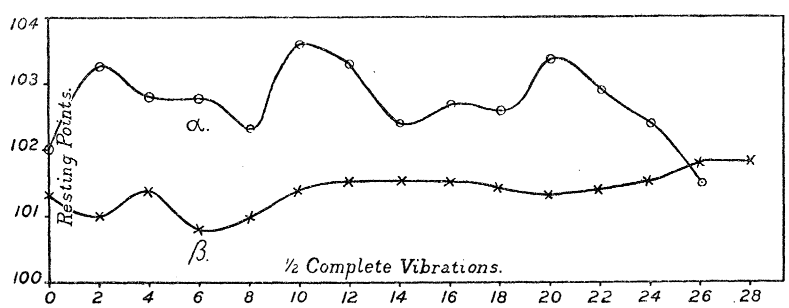


Fig III

At the conclusion of these observations the beam was fatigued for some considerable time; a second series of observations was then taken, and the successive values for the R.P., shown in curve β (Fig. III.), deduced. The second curve shows that the beam had been reduced to an almost perfectly normal condition for the load (10 gr.) used. With this particular load, and with this alone, similar results could be obtained at any time, provided a sufficient period had been allowed for the beam to recover from the strains set up during any preceding experiment.

The slight upward trend of the curve β is due to a small change which took place in the temperature (*vide infra*) during the observations.

The load which is just sufficient to produce the changes which we have here discussed, may be regarded as a *critical load.* We think it is highly probable that for many balances of certain types there are similar critical loads. In refined weighing this is a point of some importance, and calls for careful attention on the part of the observer.

During the investigation it was observed that some three or four different modes of securing the knife-edges to the beam have been adopted by various makers. Three of these are outlined in Fig. IV. Balances having their terminal knife-edges secured

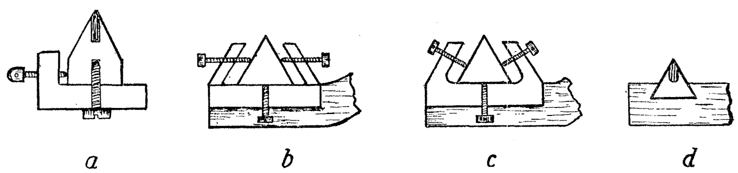


Fig IV

as in *a* appeared to be the more reliable. In this case the knife-edge is formed along the upper extremity of an agate plate, which is sunk and cemented into a brass block. This block is fastened to the beam by screws which enter from beneath; the tips also of the horizontal screws bite into and so firmly grip the block.

The curves shown in Fig. III. were drawn from data obtained when experimenting with a balance the terminal knife-edges of which were secured to the beam after the manner indicated in *b*.

This method is, in our opinion, faulty and objectionable. The solid agate prism is simply held in position by adjusting screws which can be brought to bear upon but cannot possibly grip so hard a substance as agate. As no screws enter the knife-edge prism, the prism cannot seriously be regarded as forming a definite and incorporated portion of the beam itself.

We observe, further, that the direction of the upper adjusting screws is incorrect. When the beam assumes an inclined position during weighing, there will be a tendency on the part of the loaded pans to thrust the two terminal knife-edges in one direction along the inclined plane. The slender and horizontally placed adjusting screws will be unable to completely resist the wedge-like action of the agate prisms; the screws will be bent upward and, consequently, one arm will be shortened and the other lengthened. The imperfection may be removed by directing the screws along lines which meet the sloping faces of the prisms at right angles, as shown in *c*. The forces that the screws are called upon to resist then act along their axes, and there will be no appreciable flexure.

The simple method shown in *d* cannot be recommended. To be efficient and trustworthy it demands that the prism shall exactly and rigidly fit the triangular aperture prepared for it. It is unnecessary to dilate upon the difficulties inseparable from such a scheme. We may, however, observe that, when we were using a balance in which this mode (*d*) of attaching the knife-edges to the beam was adopted, considerable irregularities in the R.P. curves were not infrequently exhibited.

(4) *Of the Effects produced by Variations in the Temperature of the Beam.*

We now proceed to examine the effects which may be produced by (1) slight differences in the temperature of the two arms of the balance, and (2) a strictly uniform increase or decrease in the temperature of the whole beam.

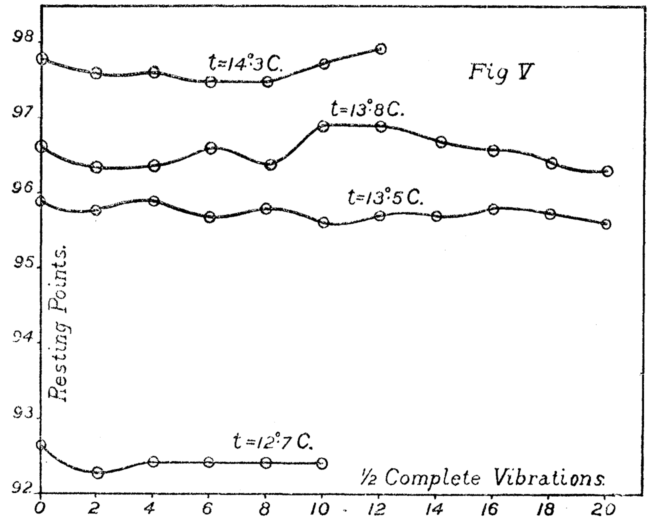
(1) If the beam is of brass, we may assume a coefficient of expansion equal to 0·00002. Taking the length of each arm as 7 cm., we expect to have, at any uniform temperature, 1 : 1 as the ratio between the two lengths. But if the temperature of the one arm differs from that of the other by so much as 1/100° C., then the length of the arm having the higher temperature will be 7·0000014 cm., and the new ratio 1 : 1·0000002. When using a sensitive balance under the conditions already described, no difficulty would be experienced in detecting so small a change. This will be apparent from the following example: The arms of a balance, the sensibility of which was = 70 for 1 mgr., were compared. Upon reversing the weights, the change in the R.P. was found to be 42. The load in each pan was 200 gr. From these data we find the ratio of the arms to be 1 : 1·0000008. If the ratio had been 1 : 1·0000002, the total change in the R.P. would have approximated 10, a quantity which could have been observed and measured with ease.

If the balance is of the long beam type, a slight difference in the temperature of the two arms would, in all probability, often exist; for the longer the beam, the greater will be the difficulty experienced in maintaining uniformity of temperature within the case.

These considerations led us to the inevitable conclusion that for weighings of the highest degree of accuracy, the total mean difference between the temperature of each arm must be much less than 1/100° C.

(2) During this research it was observed that a balance, which was being kept loaded for a lengthy period, gave different values for the R.P. from day to day. On comparing these with the corresponding thermometer readings, it was discovered that the variations were intimately connected with the rise and fall in the temperature. The related changes in the R.P. and temperature are shown graphically in Fig. V. Here the different positions of the curves evidently indicate a uni-directional

displacement of the R.P. with increase in temperature. It was accordingly decided to investigate this point more fully, and for this purpose the following method of procedure was adopted.

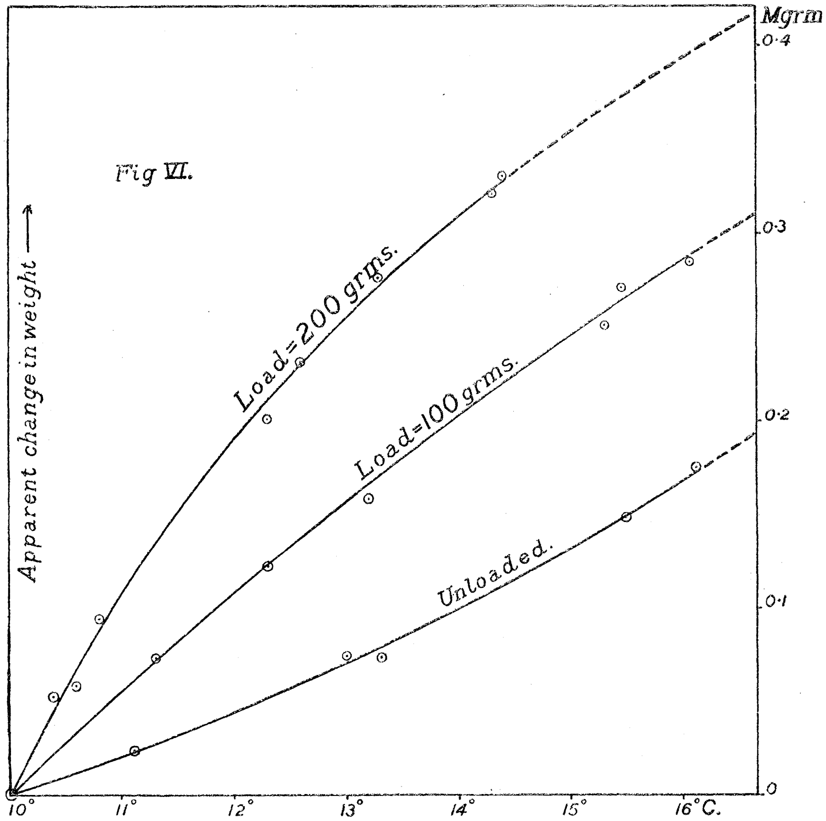


The beam was first thoroughly fatigued with the load which it was desired to use in the experiment, and so reduced to a normal condition. When the temperature within the balance case was found to be constant, observations were made for the determination of the R.P. for that particular temperature. The room was then warmed about 2° C., and as soon as the temperature had again become constant, the R.P. was re-determined. Proceeding in this manner, the R.P. was measured over a range of 5 or 6° C. Finally, the room was cooled down to approximately the initial temperature, and the first set of observations repeated.

This last constitutes a very necessary precaution, for it may happen that appreciable particles of dust may find their way more freely to one pan than to the other; the real values for the R.P. would, as a consequence, be more or less masked. If the two sets of observations effected at the lowest temperature were in close agreement, we felt justified in accepting the whole series. Upon one or two occasions only was it found necessary to discard a series owing to a want of agreement between the first and last experiments.

Using the Gallenkamp balance, experiments were made with three different loads: (a) with pans alone, (b) pans+100 gr., and (c) pans+200 gr., the last named being the maximum load for which the instrument was built. The results obtained are plotted so that the curves show the apparent increase in the value of one of the loads relatively to the other.

The resultant graphs (Fig. VI.) reveal three important points: (a) for any given load the R.P. can only be constant when the temperature is invariable; (b) the displacement of the R.P., due to a given alteration in the temperature, increases with the load; (c) for this particular balance, the curve representing the changes which occur when small loads are used is concave: with a load of about 50 or 60 gr. the concave form gives place to a straight-line curve; and lastly, as the load is increased, the type of the graph passes over into a form which is convex, the convexity increasing with the load.



A formula of the type

$$M = M_1 (1 \pm \alpha t \pm \beta t^2)$$

enables us to calculate, in each case, the apparent changes in weight at any temperature within the limits given.

In the following table we give the determined values for α and β for the several loads:—

TABLE III.—Temperature Coefficients for the Gallenkamp Balance.

Load.	α.	β.
gr. 0–5	+0·000019	+0·0000016
60	+0·000038	{ This was obtained by interpolating.
100	+0·000052	−0·00000034
200	+0·000124	−0·0000131

Very similar results were likewise obtained with the other balances. For example, the respective R.Ps. of the Oertling long beam and a Sartorius triangular beam were found to undergo, with variations in temperature and load, the approximate displacements set forth in Table IV.

TABLE IV.

Beam.	Load.	Mean displacement in scale-divisions for 1° C.
	gr.	
Oertling	0	0·6
,,	50	2·1
Sartorius	0	2·3
,,	100	3·4
,,	200	4·1

So far as we are able to judge, all balances are liable to be affected in this wholly unexpected manner.

Of the Variations in the Temperature within the Balance Case, as Observed with a Bolometer.

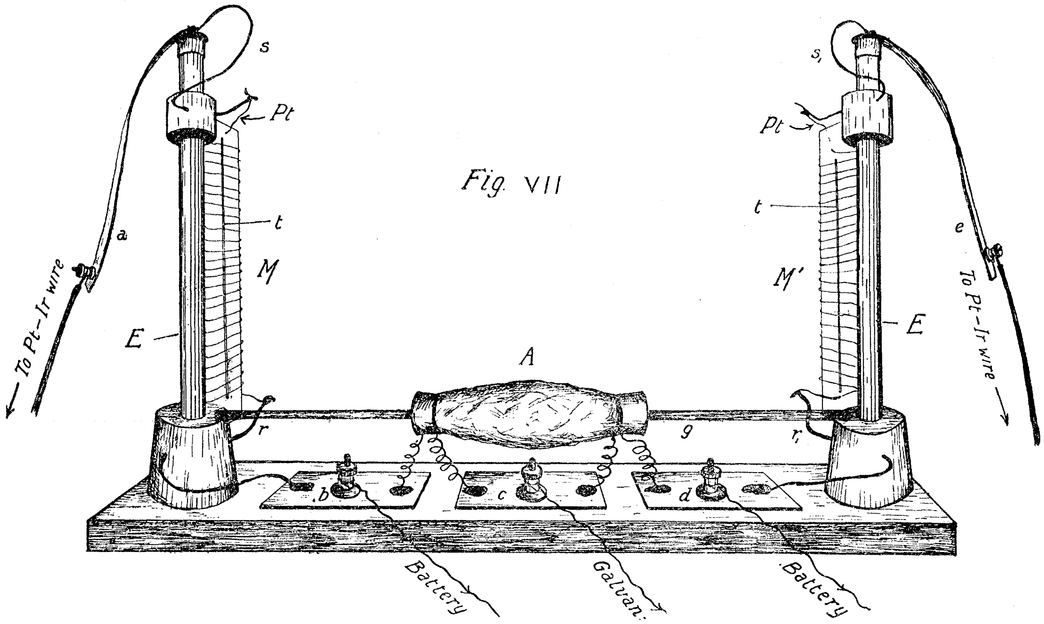
Of the numerous precautions which must be observed when it is desired to attain to a high degree of accuracy in weighing, possibly none are more important than those which tend to ensure uniformity of temperature throughout the beam.

We have already shown that any deviation from strict equality in the temperature of the two arms must, in such cases, be considerably less than 1/100° C. Now it appeared probable that small temperature differences would generally exist within a balance case, and therefore within the arms, even in spite of all the usual precautions. We therefore decided to investigate this matter somewhat minutely and endeavour to discover both their nature and magnitude under the ordinary conditions.

For this purpose a mercury-in-glass thermometer, even if constructed so as to be capable of indicating very small differences in temperature, would be wholly unsuitable on account of its sluggish nature. Hence it was decided to employ a modified form of the Langley bolometer. This was constructed in the following manner :—

Two thin rectangular sheets of mica, M, M′, each 10 cm. long and 2 cm. wide, were supported, as shown in Fig. VII., by ebonite rods, E, E, the distance between them being about 20 cm. Upon each sheet was then wound, in spiral form, about one metre of platinum wire having a diameter of 1/20 mm. ; the ends of the platinum wires were soldered to the copper wires r, s, r', s', these latter were similarly joined to the copper straps a, b, d, and e. Very fine glass capillary tubes, t, t, were thrust down on either side between the coils of platinum wire and mica. The spirals were thus made, as far as circumstances permitted, air lines.

The resistance of one of the spirals being taken as unity, that of the other was adjusted by trial until it was almost the same. The resistance of either spiral was approximately 113 ohms.



Two auxiliary coils, A, were now prepared. Each of these had a resistance of about 118 ohms, and consisted of 20 metres of insulated No. 35 German-silver wire. The two wires were wound bifilarly and together upon a short piece of thick-walled copper tubing, the surface of which had been covered with one layer of paraffined paper. After winding, the coils were soaked in molten paraffin wax and then suitably protected with silk ribbon and tinfoil; the whole was then mounted upon a glass tube, *g*, passing through the axis of the copper tube. Finally, binding screws were soldered to the copper straps *a*, *b*, *c*, *d*, and *e*, and two equal copper wire leads, twisted together, were used to join the straps, *a*, *e*, to the ends of a stretched Pt-Ir wire.* The Pt-Ir wire had an effective length of 1 metre and a diameter of 1·5 mm.

The following are the actual resistances of the four coils as determined at a temperature of 20° C.:—

Pt coil marked L = 112·77 ohms.	Auxiliary coil marked l = 117·6 ohms.
,, ,, R = 112·99 ,,	,, ,, r = 117·6 ,,

When in use the resistances of the two platinum wire spirals were adjusted to close equality by adding 0·2 ohm to the external circuit of the coil L.

It will be seen that the whole apparatus constitutes a Wheatstone's bridge of dimensions convenient for the purpose in hand, and arranged after the manner devised

* This wire was a portion of that used in a former research ('Phil. Trans.,' 1898, vol. 191, p. 378). It is remarkably uniform, and, when re-calibrated for the present investigation, was found to have a mean resistance of 0·000155 ohm per 1 mm.

by CAREY FOSTER. The two coils to be compared are the platinum wire spirals, whose temperature may vary. The whole of the bridge, with the exception of the slide wire, can be introduced into a short beam balance case.

Using an Elliott "Century" D'Arsonval galvanometer, having a resistance of 50 ohms, and a bichromate cell working through a suitably adjusted rheostat, it was found that a movement of the slider equal to 0·3 mm. could be detected with certainty.

Taking the temperature coefficient of the platinum wire spirals as 0·004, and using the data already given, it can be shown that the bolometer enabled us to measure temperature differences rather smaller than 1/10,000° C.

In the experiments which we now describe, the temperature differences were in the majority of cases recorded to within 1/5000° C. only.

Experiments with the Bolometer.

(1) The bolometer was placed inside a Bunge balance case and arranged so that the centres of the platinum spirals were as near as possible to the ends of the beam. The case was then closed and left for a time, in order that the temperature within might become normal. Bolometer readings were then commenced and continued for a period of $19\frac{1}{2}$ minutes: the observations were taken at intervals of 10 seconds, and

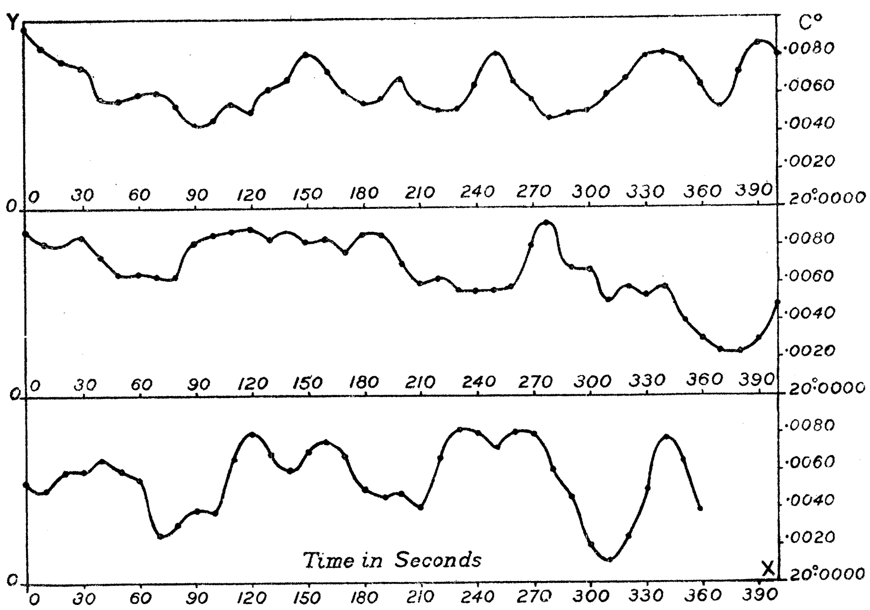


Fig VIII

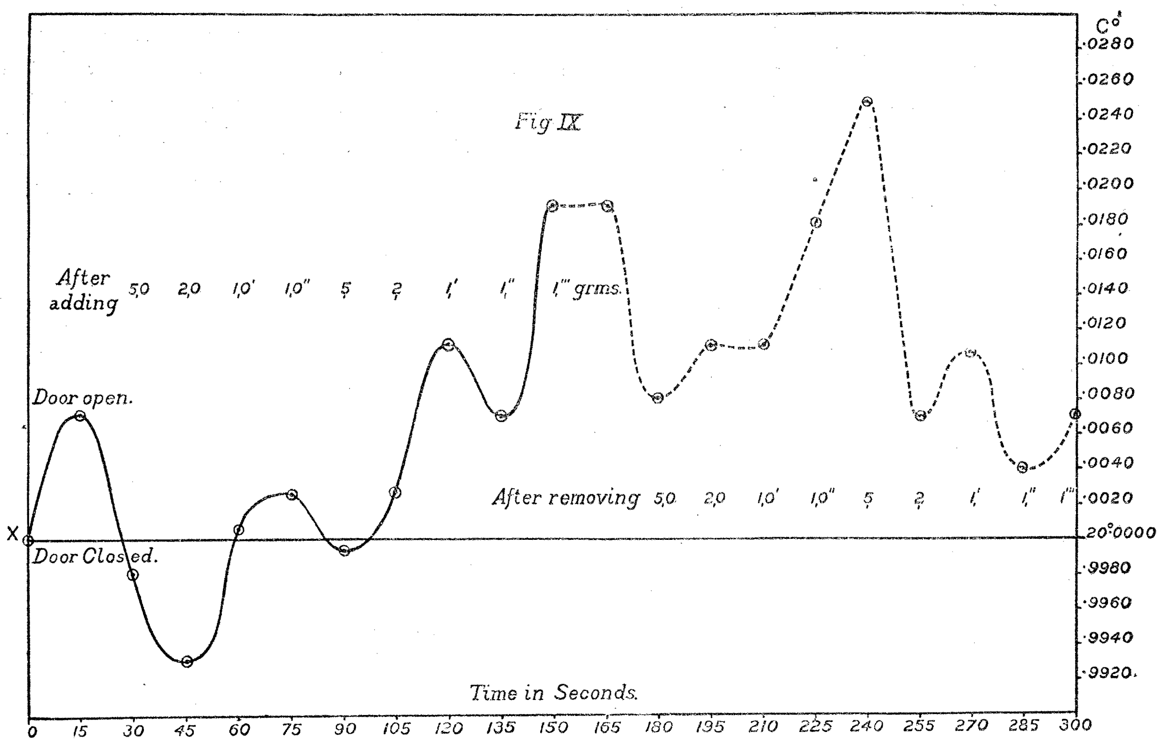
reduced to their equivalent temperature values. On plotting the temperatures along the Y-axis, and the corresponding time in seconds along the X-axis, the graph shown in Fig. VIII. is obtained. For convenience this is divided into three sections.

Experimenting in this way with the other balances, precisely similar results were obtained. We were thus led to conclude that a temperature common to the two ends of the beam seldom exists even for a few seconds. The extreme variations in the experiment cited above amounted to almost 1/100 of a degree C.

(2) We next proceeded to ascertain what the magnitude of the variations in the temperature might be during the time required for loading and unloading the pans.

The temperature within the case being as nearly as possible normal, the bolometer was read. On lifting the shutter a sudden and marked change in the temperature occurred. Nine weights were then successively placed upon one of the pans, bolometer readings being taken after the introduction of each weight. Lastly, the weights were removed one by one, and bolometer readings taken after the withdrawal of each. All the readings were then reduced, as in the preceding experiment, to their equivalent temperature values, and the results plotted in a corresponding manner.

The observed temperature changes occurring during one of a number of experiments are shown graphically in Fig. IX.



We observe that in this particular experiment the extreme difference in the temperature of the air in the vicinity of the ends of the beam was approximately 0°·025 C. But it was found that the larger disturbances just recorded gave place within a few minutes after the conclusion of the experiment, to the usual and much smaller ones; the mean temperature was, however, a little higher than at first. The slight increase in the temperature during the experiment is indicated by the general up-trend of the curve.

Of the Method adopted for Maintaining Uniformity in the Temperature of the Air immediately surrounding the Beam.

An attempt was now made to devise some method for avoiding fluctuations in the temperature of the air in the neighbourhood of the beam. Upon reflection it appeared highly probable that the greatest measure of success would be attained by (1) greatly reducing the air space surrounding the beam, and (2) enclosing the beam within a chamber consisting of materials capable of readily absorbing, reflecting, and distributing any heat radiations which, under the usual conditions, enter the beam and so become responsible for some of the errors to which attention has been directed.

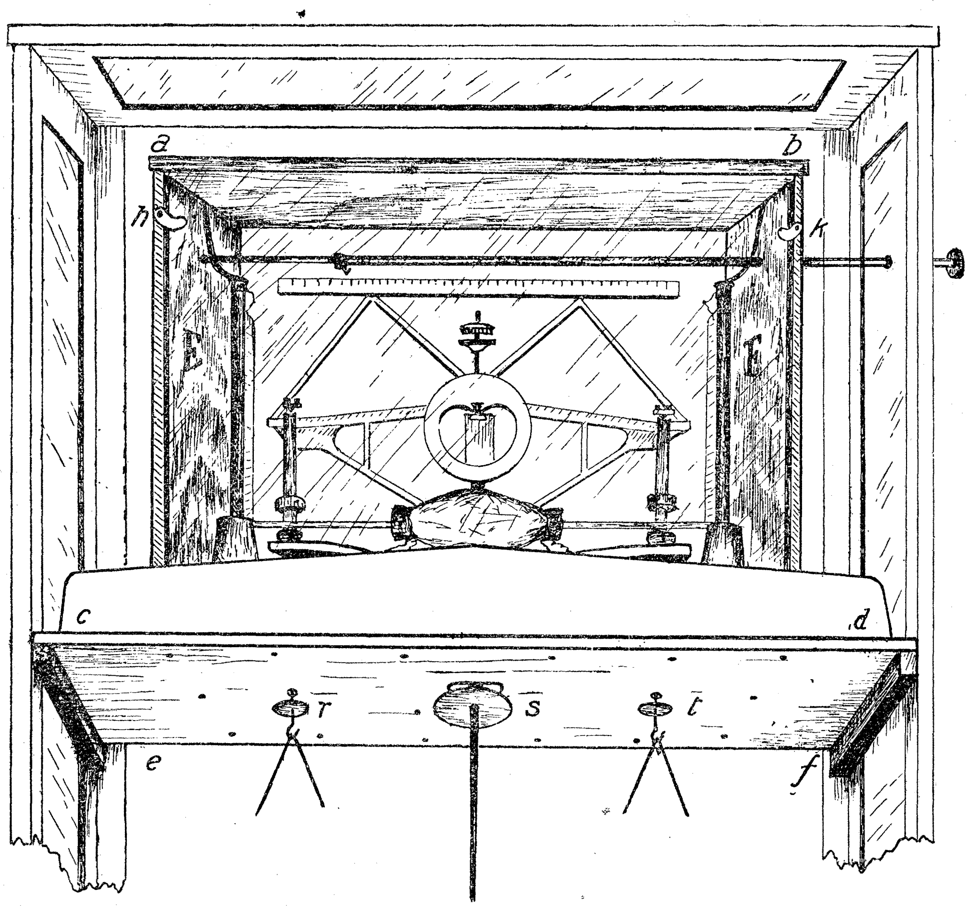


Fig. X.

Now the researches of De la Roche, Melloni, and others show that the most suitable materials for this purpose are glass, well-dried wood, and metals; and of all metals aluminium possesses in the most marked degree those characteristics that we most desire, namely, lightness, rigidity, great reflecting power, and high conductivity for heat.

Accordingly a small chamber, represented in Fig. X., was built up round the beam,* within the maker's case. The base-plate, *c*, *d*, *e*, *f*, was of aluminium, having a thickness of 2·2 mm., suitably slotted and perforated for the passage of the pointer and hooks by which the pans are suspended. The ends, E, F, were made of well-seasoned mahogany 11 mm. thick, whilst the front, back, and also the top of the chamber were of 8 mm. thick plate-glass. The front glass shutter was removable after dropping the holders, *h*, *k*. Discs *r*, *s*, *t* of thin aluminium were fastened to the pointer and pan hooks a little below the base-plate.

That this addition to the equipment of a delicate balance not only enhances the reliability of measurements made with the instrument, but also enables the observer to obtain his results with a minimum expenditure of time, is, we believe, shown by the following evidence. The bolometer was introduced into the inner or beam case, which was then closed. After a considerable interval the following experiments were made :—

Experiment I.—The ordinary sliding door of the balance case was opened and closed a number of times without producing the slightest effect upon the bolometer.

Experiment II.—Nine weights were introduced and then removed in the usual manner six times in succession. The bolometer was quite unaffected.

Experiment III.—The observer's closed hand was held close to the R pan for 5 minutes. During the first 3 minutes no visible effect was produced; but during the remaining 2 minutes the measured temperature difference within the beam case was equal to $+1/1000$ of a degree C.

Experiment IV.—This was similar to the preceding experiment, but the observer's hand was held close to the L pan. There was no observable change during the first 3 minutes, but at the expiration of the remaining 2 minutes the temperature difference amounted to slightly more than $-1/500°$ C.

Experiment V.—When the experimenter's hand was held close to the pillar and between the pans, a temperature difference amounting to 1/5000 of a degree C. was set up at the end of 5 minutes.

Experiment VI.—A sensitive mercury-in-glass thermometer was placed close beneath the aluminium base-plate and *Experiment I.* repeated. During 5 minutes the thermometer rose 1°·9 C.

The results obtained from *Experiment II.*, together with others found by repeating the observations in the absence of the beam case, are given in the form of the curves seen in Fig. XI.

These experiments show that the inner chamber is highly effective, and that its presence ensures a practically perfect uniformity of temperature throughout the entire beam.

Under ordinary conditions, and in the absence of some such device as the above,

* The balance chosen for this experiment was one by GALLENKAMP. Either of the other balances would have been equally convenient.

differences in the temperature of the two arms will and must very naturally occur. Every time a weight is introduced into the pan a current of warm air will ascend from the observer's hand; and this, on coming into contact with the arm of the beam above, will inevitably result in a relative lengthening of that arm, and a consequent

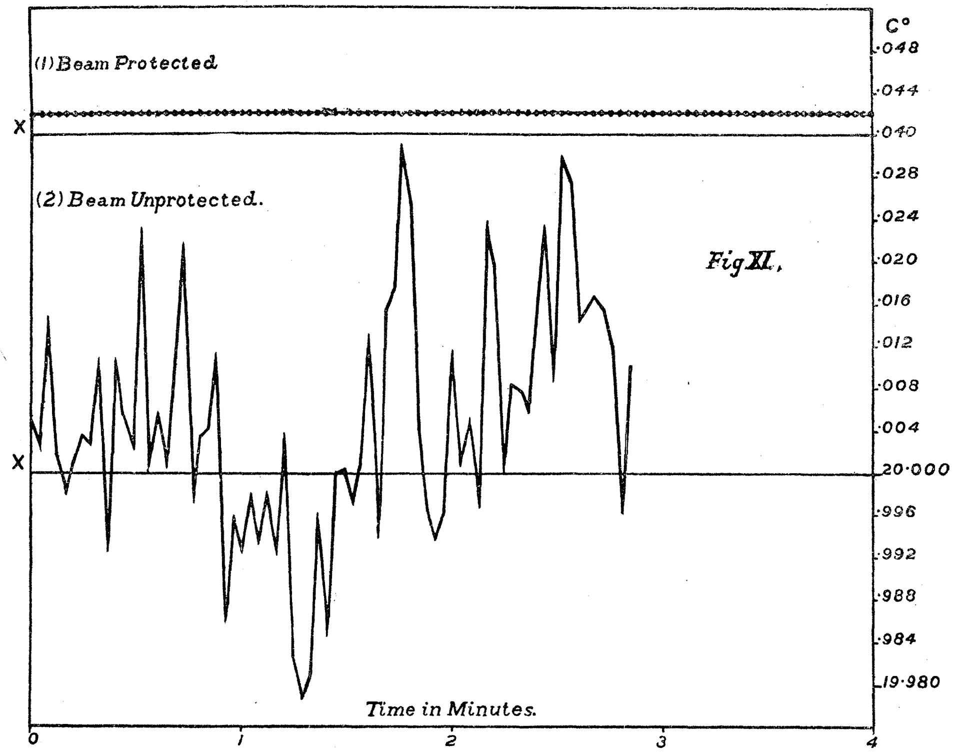


Fig. XI.

uncertainty in the true value of the weight sought. Now, although these relatively large disturbances gradually die away and finally disappear, they only do so after a considerable lapse of time; but the smaller ones, to which attention has already been directed, still remain, and may therefore introduce a very slight but perceptible degree of inaccuracy.

When, however, the beam is protected as described, it is never exposed to the open air during the operations of loading and weighing; and of the heat radiated from the observer's hand in the direction of the beam, not more than about 10 per cent. will enter the highly reflecting base-plate. The heat-rays that actually enter the base-plate will be distributed within the metal in all possible directions with great rapidity; hence the small radiations from the upper surface of the plate towards the beam will be approximately uniform; and therefore the possibility of appreciable unequal heating of the two arms of the balance is precluded.

The base-plate itself guards the beam almost completely against the effects of any ascending convection currents. But the necessary holes for the passage of the pan suspensions and pointer render possible the introduction of an error from this source.

It was with the object of minimising the slight disturbances that might be thus produced, that the discs r, s, t (Fig. X.) were added. These intercept and deflect the convection currents away from the openings and cause them to expend their energy upon the solid plate.

Of the Minor Undulations to be found in a Resting-Point Curve.

We are now in a position to determine experimentally whether the minor and persistent sinuosities found in nearly all our R.P. curves are due to temperature fluctuations more or less periodic in their nature.

We know that a periodic variation in the temperature at one end of a bar will give rise to heat-waves which will journey towards the other end. If the oscillations in the temperature take the form of a simple harmonic motion, then for a homogeneous and uniform bar the temperature variations will be correctly represented by a curve of sines. As successive heat-waves follow each other, the bar will be alternately lengthened and shortened; and if the variations in length be represented graphically, we again obtain a curve of sines.

Now, any appreciable changes in the relative lengths of the two arms of a balance, whether harmonic or not, should manifest themselves by corresponding variations in the R.P. Hence, from the form of any given R.P. curve, we ought to be able to draw a fairly correct inference as to the nature of the heat-waves that may be travelling along the beam at the time of weighing.

Conversely, if it can be shown that the temperature of the beam remains strictly uniform during the operations of weighing, then the R.P. curve for a perfectly fatigued balance will be a right line, horizontal in direction.

This theory was tested by means of the protected Gallenkamp balance, which for the purpose was loaded with 20 gr. (Any other load would have been equally convenient.) The beam was released and observations commenced and continued, without any preliminary fatiguing of the beam, until sufficient data had been acquired for 19 independent determinations of the R.P. The results obtained in this experiment are shown in the curve, Fig. XII.

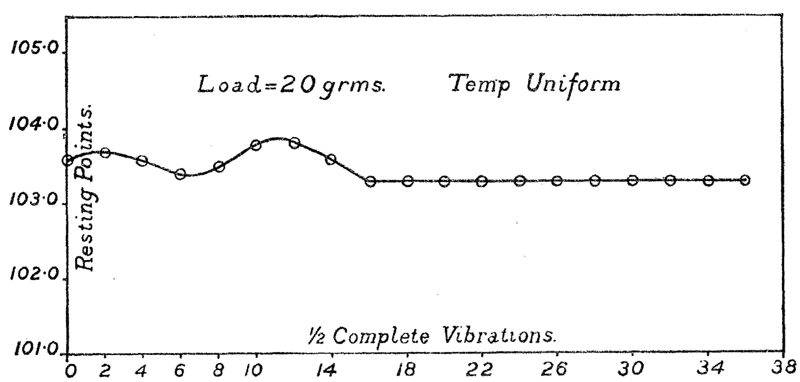


Fig XII.

The small undulations which occur in the first part of this curve cannot be due to flexure of the beam, for it was found that the sensibility of this particular balance was precisely the same for all loads. Neither could they have been caused by fluctuations in the temperature, for those would have been detected by the bolometer.

The observed variations therefore probably owe their origin, as we have already suggested, to some slight relative movement of the agate knife-edges consequent upon the loading of the beam. The correctness of this view may also be inferred from the fact that when the above experiment is repeated with the balance duly fatigued, the obtained R.P. curve is a right line wholly free from undulations.

We may therefore conclude that the slight periodic variations which the bolometer has shown to exist in the temperature within the usual balance case, are responsible for the more or less harmonic motions recorded in the R.P. curve; and it appears very probable that these effects must always be detectable in a sensitive balance, the beam of which is not guarded in some such manner as we have indicated.

Of the Major Undulations to be Found in a Resting-Point Curve.

It may be observed that when an experienced worker is using a balance, he places the required weights upon the pan at approximately equal intervals of time. As each weight is introduced the currents of warm air rising from the observer's hand will envelope the end of the beam above; these currents will produce heat-waves, roughly harmonic, within the beam.

The magnitude of these temporary heat-waves will be considerable when compared with the magnitude of those which are usually flowing along the beam. As soon as the loading is ended and the shutter dropped, the larger waves will begin to die away, and after a short time disappear. But whilst they remain their existence is proved and their influence indicated, by the exceptionally large undulations which appear in the majority of the R.P. curves drawn from data obtained immediately after the loading of the beam has been completed.

It may be noted in passing that the first portions of such R.P. curves which we have often obtained, forcibly remind one of the graphs representing the results of the experiments of DESPRETZ, FORBES, and others, upon the heat conductivity of uniform bars, and it is not improbable that further investigation would show the curves to be logarithmic in form.

Experience has taught us that the particular source of error in weighing which we are now discussing, is sufficiently guarded against by allowing a period of about 10 minutes to elapse between the completion of the loading and the actual weighing. And if the balance is fatigued during this same time, we also avoid the error which appears to be due to the slight initial movements in the several groups of adjusting screws. These precautions are now almost invariably taken by observers who are here engaged in researches for which very accurate weighing is essential.

Of Certain Experiments which were made to Test the Efficacy of the Precautions advocated.

Before concluding we would briefly allude to a few experiments of a comparative nature. These were performed by means of the Oertling long beam and protected Gallenkamp balances. Owing to the extremely low value of the temperature coefficient of the former, and the almost complete freedom from strain effects of the latter, these balances were particularly appropriate for the object in view.

It was desired to ascertain how closely the results obtainable with two balances of totally different types might agree when the precautions, which we now know to be necessary, are taken. For this purpose a block of brass, the mass of which was approximately 100 gr., was weighed at various times on successive days with the screened and fully fatigued Oertling balance. The same block was then weighed a number of times with the other balance.

The results obtained are set forth in Table V. (see next page).

As the R.P. coefficient of our Oertling balance was insignificant, the observed values for the mass weighed have not been reduced to a common temperature. Such a procedure would, in the present instance, have introduced a correction considerably smaller than the possible experimental errors due to other causes.

We note (1) that the greatest difference in the four daily mean values obtained with the Oertling beam are 0·10, 0·07, 0·17, and 0·03 mgr. respectively; these are irregular when referred to the corresponding thermometer readings, and are probably chiefly due to some minute and corresponding variations in the temperature of the two arms of the beam. Such irregularities cannot be completely avoided unless the beam is protected in the manner already described. The difference in the values yielded by the protected Gallenkamp beam was 0·02 mgr. (2) That the two final mean values for the weight W of the block are—

By the Oertling	W	= 100·007065 gr.
,, Gallenkamp	,,	= 100·007064 ,,

The difference here observed is not greater than $1/10^7$ of the whole.

This remarkably close agreement affords still further proof of the effectiveness of the precautions which are, in our opinion, necessary when a very high degree of accuracy in weighing is desired.

We may add that although the values given in Table V., and obtained by means of the protected balance, show amongst themselves a maximum difference of 0·02 mgr., yet further weighing proved that, when the beam was first fully fatigued, the observed differences in the values were, in general, not greater than some few thousands of a milligramme. The correctness of this view is likewise supported by the results represented in the latter portion of the curve in Fig. XII.

TABLE V.

Date.	Time.	Temperature within balance case.	Observed weight of block.	Mean values.	Maximum differences.
		° C.	Grm.	Grm.	Grm.
Oertling Balance.					
September 24, 1909	10.20 a.m.	16·2	100·00702	100·00704	0·00014
	12.40 p.m.	15·7	713		
	1.15	15·8	699		
	2.50	15·9	703		
September 25, 1909	10.20 a.m.	16·1	707	100·00706	0·00007
	10.40	16·1	702		
	11.35	13·7	707		
	12.50 p.m.	14·0	709		
September 27, 1909	11.15 a.m.	14·2	707	100·00706	0·00017
	11.30	14·4	697		
	Noon	14·4	707		
	12.15 p.m.	14·5	703		
	12.30	14·6	707		
	1.0	14·6	714		
September 28, 1909	9.58 a.m.	13·4	710	100·00710	0·00003
	10.5	13·5	710		
	10.12	13·6	712		
	10.18	13·6	711		
	10.25	13·6	710		
	10.30	13·6	709		
			Final mean = 100·007065 gr.		
Protected Gallenkamp Balance.					
September 28, 1909	11.40 a.m. to 12.34 p.m.	14·0	100·00707	100·007064	0·00002
		·0	707		
		·0	707		
		·0	705		
		13·9	706		
		14·0	707		
		·0	707		
		·0	705		

Difference between the two final means = 0·000001 gr.

The comparative experiments, the results of which are given in the three curves shown in Fig. XIII., clearly indicate the advantages to be derived from the use of the inner protecting case and accompanying bolometer.

Curve I. is a record of the variations in the temperature upon one occasion in the absence of the beam case. Curve II. was obtained after the beam case had been introduced. And, although the graph roughly approximates a right line, we learn from its direction that a suitable position had not been chosen for the balance; the instrument had been placed unsymmetrically with regard to the radiant energy falling upon it. From Curve III. we infer that the new position which the balance occupied during the experiment was perfectly satisfactory and almost ideal.

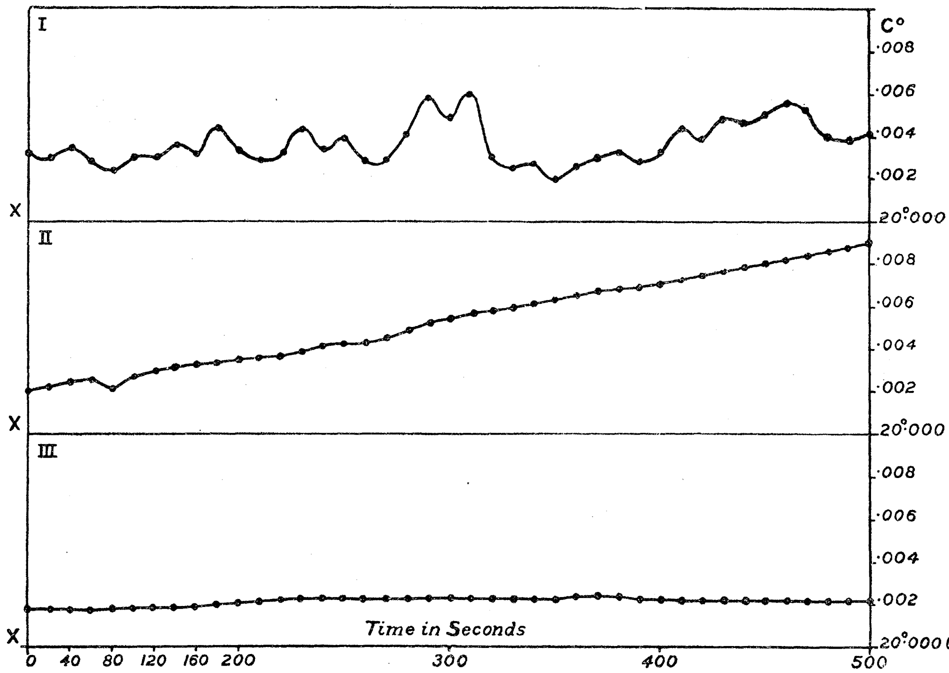


Fig XIII

We have deliberately refrained from introducing into this paper numerous and lengthy tables of data. In place of such tables, which, in our opinion, could serve no useful purpose, representative graphs of typical cases have been inserted. These have the merit, not always possessed by columns of data, of revealing, almost at a glance, that which it is desired to convey. It is obviously impossible and undesirable to give more than a very few of the somewhat large number available for our purpose.

In conclusion, the author would express his indebtedness to the Government Grant Committee of the Royal Society for the continued loan of certain apparatus used in connection with this research. He would also acknowledge his obligations to Mr. W. W. Fisher, of the Oxford University Chemical Laboratory, Messrs. Baird and Tatlock, and Messrs. A. Gallenkamp and Co., all of whom very kindly granted him the loan of some of their finest balances. Without their generous aid it would have been exceedingly difficult to bring certain portions of this investigation to a satisfactory termination. Finally, the writer's best thanks are due to Mr. R. T. Günther, Fellow of Magdalen College, Oxford, for the valuable help and advice which he has received from him.

Summary.

The experiments detailed in this paper indicate that the majority of delicate balances are liable to be adversely affected (1), by changes in the temperature of the neighbouring air and (2), by certain slight inherent, and possibly unavoidable, imperfections in the structure of the beam.

(1) The errors due to the first-named cause may be introduced in three different ways :—

(*a*) By a relatively large but very unequal increase in the temperature of the two arms of the beam during the operation of loading (Fig. XII.). The error arising from this may be avoided by enclosing the beam within an extra case of metal and glass (Fig. X.). In the absence of a protecting case, an interval of from 10 to 15 minutes should be allowed to elapse between loading and the final determination of the weight.

(*b*) From a want of symmetry in the position of the balance with regard to the radiant energy entering by the window and emanating from the walls of the room (Curve II., Fig. XIII.).

The best position for a balance in a given room can only be determined by trial. When a suitable differential bolometer is available very little difficulty need be experienced in selecting the most favourable spot.

(*c*) By any strictly uniform change in the temperature of the whole beam.

If the balance is not kept at one temperature it will be necessary, in order to secure the highest possible degree of accuracy, to know the R.P. temperature coefficient for any given load. It may be convenient to determine once for all the R.P. temperature coefficients for some three or four suitably chosen loads; the corresponding values for other loads can then be found with sufficient accuracy by interpolation.

(2) The errors which may be introduced through slight imperfections in the beam may be evaded by first counterpoising the object to be weighed with the required standards and then leaving the beam to oscillate for about 10 minutes. By this device the beam is fatigued and reduced to a normal condition. The actual weighing may then be accomplished.